ABSTRACT
A GENERALIZED FRAMEWORK FOR SINGULAR SPECTRUM ANALYSIS: DATA-ADAPTIVE AND REGULARIZED EXTENSIONS

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Singular Spectrum Analysis (SSA) has emerged as a powerful nonparametric tool for time series analysis, enabling decomposition, reconstruction, and forecasting of structured signals without reliance on strong model assumptions. However, traditional SSA frameworks face notable limitations when applied to high-dimensional, noisy, and functional time series data commonly encountered in modern applications. This dissertation presents a comprehensive generalization of SSA through the development of a unified Hilbert space-based framework that systematically integrates both data-adaptive and regularized extensions.

The proposed framework, termed Hilbert Space SSA ($\mathcal{H}$SSA), extends classical SSA by embedding time series in Hilbert spaces, thus encompassing scalar, multivariate, and functional data representations within a cohesive mathematical structure. Within this framework, the dissertation introduces the Data-Adaptive SSA ($\mathcal{D}$SSA), which enhances decomposition efficiency by adaptively selecting trajectory subspaces tailored to the structure of the data. To address the challenges posed by noise and overfitting, a Regularized $\mathcal{H}$SSA (r$\mathcal{H}$SSA) is developed by incorporating smoothness-inducing penalties into the singular value decomposition process, yielding robust low-rank approximations suitable for high-frequency or ill-posed scenarios.

The theoretical developments are supported by extensive simulation studies and empirical applications, including the analysis of call center and satellite image data, which demonstrate the superior reconstruction accuracy and computational gains of the proposed methods. Forecasting strategies are also extended under the generalized $\mathcal{H}$SSA paradigm, enabling effective prediction of both scalar and functional time series components. Collectively, this dissertation advances SSA theory and practice by establishing a scalable, interpretable, and robust analytical toolkit for complex time-dependent data, with significant implications for fields ranging from signal processing and climatology to finance and biomedical sciences.